**FEDERAL PUBLIC SERVICE COMMISSION**
**COMPETITIVE EXAMINATION-2021**
**FOR RECRUITMENT TO POSTS IN BS-17**
**UNDER THE FEDERAL GOVERNMENT**

**Roll Number**

# PURE MATHEMATICS

| TIME ALLOWED: THREE HOURS | MAXIMUM MARKS = 100 |
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**NOTE:**
- **(i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
- **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- **(iii)** Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed.**

## SECTION-A

**Q. 1.** **(a)** Let $\Psi$ be a homomorphism of group G into group $\tilde{G}$ with kernel K, prove that K is a normal subgroup of G. (10)

**(b)** Prove that if H and K are two subgroups of a group G, then HK is a subgroup of G if and only if HK=KH. (10) **(20)**

**Q. 2.** **(a)** Find elements of the cyclic group generated by the permutation. (10)

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 2 & 6 & 1 \end{pmatrix}$$

**(b)** (10) **(20)**
Verify that the polynomials $2-x^2$, $x^3-x$, $2-3x^2$ and $3-x^3$ form a basis for the set $P_3(x)$; the set of all polynomials of degree three. Also express the vectors $1+x^2$ and $x+x^3$ as a linear combination of these basis vectors.

**Q. 3.** **(a)** Let V be the real vector space of all function from R to R. Show that $\{\cos^2 x, \sin^2 x, \cos 2x\}$ is linearly dependent while { cosx, sinx, coshx, sinhx} are linearly independent. (10)

**(b)** Solve the system of linear equations: (10) **(20)**

$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 + 11x_3 + 8x_4 &= 7 \end{aligned}$$

## SECTION-B

**Q. 4.** **(a)** If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$. (10)

Show that $\frac{\partial^2 f}{\partial y \partial x}(x,y) = \left(\frac{x^2 - y^2}{x^2 + y^2}\right)$

**(b)** Evaluate $\int_0^6 f(x)dx$ where $f(x) = \begin{cases} x^2 \text{ when } x < 2 \\ 3x - 2 \text{ when } x > 2 \end{cases}$ (10) **(20)**

**Q. 5.** **(a)** Let $I_n = \int_0^\infty x^n e^{-x} dx$ where $n$ is an integer. Prove that (10)

$I_n = n\, I_{n-1}$ Hence show that $I_n = n!$

**(b)** **i.** Write $r = \dfrac{8}{2 - \cos\theta}$ in rectangular coordinates. (10) **(20)**

**ii.** Write $x^4 + 2x^2y^2 + y^4 - 6x^2y + 2y^3 = 0$ in polar coordinates.

**Q. 6.** **(a)** Evaluate $\iint_D dydx$ and $\iint_D dxdy$ where $D$ is the region bounded by the y-axis, the lines x=2 and the curve $e^x$. (10)

**(b)** Investigate the curve $y = \dfrac{x^3 - x}{3x^2 + 1}$ for points of inflexion. (10) **(20)**

## SECTION-C

**Q. 7.** **(a)** Sum the series $1 + \frac{1}{2}\cos\theta + \frac{1.3}{2.4}\cos 2\theta + \frac{1.3.5}{2.4.6}\cos 3\theta + \ldots$ (10)

**(b)** Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$ (10) **(20)**

**Q. 8.** **(a)** Construct the analytic function $f$ whose real part is $U = x^3 - 3xy^2 + 3x + 1$ (10)

**(b)** Evaluate $\int_C \dfrac{dz}{z^2 + 2z + 2}$ Where C is a square with corners (0,0),(-2,0),(-2,-2) and (0,-2). (10) **(20)**

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