FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-I

Q.No.1.
(a) Let H be a subgroup of a group G. Prove that the normalizer of H in G (i.e. $N_G(H)$) is a subgroup of G. — 10
(b) Prove that a group of prime order is cyclic. — 10

Q.No.2.
(a) Write three non-isomorphic groups of order 12. — 10
(b) Prove that a group G is isomorphic to a subgroup of group of automorphisms of G. — 10

Q.No.3.
(a) Construct Cayley's table for Multiplication Modulo 7 of — 8 (4+3+1)
$$Z_7 - \{0\} = \{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{4}, \bar{5}, \bar{6}\}.$$
Show that Z_7 is an integral domain. (You may use Cayley's table.)
Is Z_7 a field? Justify your answer.
(b) Give an example of zero divisor in $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{4}, \bar{5}\}$. — 5 (2+1+2)
Is Z_6 an integral domain? Justify your answer.
(c) What is FIELD EXTENSION? — 7
Verify that the field $\mathbb{Q}[\sqrt{5}] = \{x + y\sqrt{5} : x, y\ \mathbb{Q}\}$ is an extension of $\mathbb{Q}$.

Q.No.4.
(a) Show that $W = \left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R}\right\}$ is a subspace of the vector space $\mathcal{M}_2(\mathbb{R})$ consisting of all 2×2 matrices over $\mathbb{R}$. — 10
(b) Prove that if a subset $\{v_1, v_2, \cdots, v_k\}$ of a vector space V is linearly dependent then one vector among $v_1, v_2, \cdots, v_k$ is linear combination of the remaining vectors. — 4
(c) (1) What is dimension of $\mathbb{R}^3$. — 6 (1+1+2+2)
(2) Write a basis of $\mathbb{R}^3$.
(3) Is $\{(1,1,0), (1,1,2), (1,0,1), (0,1,2)\} \subseteq \mathbb{R}^3$ linearly dependent or independent? Justify your answer.
(4) Is $\{(0,0,0), (1,1,2), (1,0,1)\} \subseteq \mathbb{R}^3$ linearly dependent or independent? Justify your answer.

Q.No.5. (a) Define eigen value of a square matrix. 10

Find eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) Find reduced echelon form of the matrix 10

$$A = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 1 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

(c) Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be eigen values of a square matrix $A = [a_{ij}]$. What are $|A|$ and $trace(A)$ in terms of λ_i's?

SECTION-II

Q.No.6. (a) Find equations of tangent plane and normal line at a point (x_1, y_1, z_1) of ellipsoid 10

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

(b) Find equation of the ellipse centered at the origin, a focus at (3, 0) and vertex at (3, 0). 5

(c) Find the polar equation of a parabola $x = 8y^2$. 5

Q.No.7. (a) Find the equation of elliptic paraboloid

$$x = y^2 + z^2$$

In spherical coordinates.

(b) Convert the following equation of quadratic surface to standard form. What is this surface? 10

$$4x^2+y^2+4z^2-16x-2y+17=4$$

Q.No.8. (a) Find curvature of the space curve 10

$$\vec{r}(t) = 2t\hat{i} + t^2\hat{j} + \frac{1}{3}t^3\hat{k}$$

(b) (1) Find first fundamental form of the surface 10 (6+4)

$$\vec{r}(u, v) = (\cos u, \sin u, v)$$

(2) Write formulae for normal and Guassian curvature of a surface

$$\vec{r} = \vec{r}(u, v)$$

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PURE MATHEMATICS, PAPER-II

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NOTE: **(i)** Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
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SECTION-I

Q. No. 1. (a) Use the Mean Value Theorem to show that **(10)**
$|\sin x - \sin y| \le |x - y|$
for any real number x and y.

(b) Use Taylor's Theorem to prove that **(10)**
$\ln\sin(x+h) = \ln\sin x + h\cot x - \frac{1}{2}h^2\csc^2 x + \frac{1}{3}h^3\cot x\csc^2 x + \cdots.$

Q. No. 2. (a) Evaluate $\lim_{x\to 0} \frac{\sin x - \ln(e^x\cos x)}{x\sin x}$. **(8)**

(b) Find the equation of the asymptotes of $2xy = x^2 + 3$. **(6)**

(c) Evaluate the integral $\int_0^2 x^3(\sqrt{2x+3})\,dx$. **(6)**

Q. No. 3. (a) Verify that $f_{xy} = f_{yx}$ for the following function: **(8)**
$f(x,y) = e^{xy}\cos(bx+c)$.

(b) Find the points of relative extrema for $f(x) = \sin x\cos 2x$. **(6)**

(c) Evaluate the limit $\lim_{x\to 0}\frac{1-\cos x}{x^2}$. **(6)**

Q. No. 4. (a) Let $d: X\times X \to R$ be a metric space. Then $d': X\times X \to R$ defined by **(10)**
$d'(x,y) = \frac{d(x,y)}{1+d(x,y)}$
is also a metric.

(b) Show that an open ball in metric space X is an open set. **(5)**

(c) Show that convergent sequence in a metric space is Cauchy sequence. **(5)**

Q. No. 5. (a) Let (X,d) be a metric space, a subset A of X is dense if and only if A has non-empty intersection with any open subset of X. **(8)**

(b) Determine whether the given series converges or diverges:
$\sum_{1}^{\infty}\frac{(2n)!}{4^n}$ **(6)**

(c) Determine whether the given series converges absolutely, converges conditionally or diverges:
$\sum_{1}^{\infty}\frac{(-1)^n n!}{(2n)!}$. **(6)**

PURE MATHEMATICS, PAPER-II

SECTION-II

Q. No. 6. (a) Use De Moivre's Theorem to evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. **(10)**

(b) Evaluate $\oint_C \frac{z+2}{z}dz$, where C is the circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$. **(10)**

Q. No. 7. (a) Find the Laurent series that represents the function: **(10)**

$$f(z) = z^2 sin\left(\frac{1}{z^2}\right).$$

(b) Evaluate the sum of the infinite series: **(10)**

$$cos\theta - \frac{1}{2}cos2\theta + \frac{1}{3}cos3\theta - \frac{1}{4}cos4\theta + \cdots.$$

Q. No. 8. (a) Find the Fourier transform of : **(10)**

(i) $f(t) = e^{-|t|}$ (ii) $f(t) = sin\alpha t^2$

(b) Find the residue at $z = 0$ of the functions: **(10)**

(i) $f(z) = \frac{1}{z+z^2}$ (ii) $f(z) = zcos\left(\frac{1}{z}\right)$