FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

PURE MATHEMATICS, PAPER-I

Time Allowed: 3 Hours | Maximum Marks: 80

Note: (i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q.** Paper.

(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** question from **SECTION-B. ALL** questions carry **EQUAL** marks.

(iii) Extra attempt of any question or any part of the attempted question will not be considered.

(iv) Use of Calculator is allowed.

SECTION-A

Q.1. **(a)** For any integer n let $a_n : Z \to Z$ by such that $a_n(m) = m + n$, $m \in Z$.

Let A = $\{a_n ; n \in Z\}$. Show that A is the group under the composition of mappings.

(b) Show that the group of all inner automorphisms of a group G is isomorphic to the factor group of G by its center.

Q.2. **(a)** Let A and B be cyclic groups of order n. Show that the set Hom(A.B) of all homomorphisms of A to B is a cyclic group.

(b) Prove that group G is abelian iff G/Z(G) is cyclic, where Z(G) is Centre of the group.

Q.3. **(a)** Define the dimension of a vector space V, prove that all basses of a finite dimension vector space contain same number of elements.

(b) Show that the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent.

Q.4. **(a)** The set $\{v_1, v_2, \ldots\ldots, v_n\}$ of vectors is a vector space V is linearly dependent if and only if some v_i is the linear combination of the other vectors.

(b) Let A, B be two ideals of the ring R. Then show that

$$\frac{A+B}{A} \cong \frac{B}{A \cap B}.$$

Q.5. **(a)** If A is $n \times n$ matrix then

(i) Determinant of $(A-\lambda I)$ where λ is a scalar in a polynomial $P(\lambda)$.

(ii) The eigenvalues of A are the solutions of $P(\lambda) = 0$.

(b) If A is an ideal of the ring R with unity such that $1 \in A$, then $A = R$

SECTION-B

Q.6. **(a)** Find an equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are given. Hence find equations of the tangent and normal at any point 'θ' on the ellipse.

(b) Prove that an equation of the normal to the asteroid

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

is $x \sin t - y \cos t + a \cos 2t = 0$, t being parameter.

Q.7. **(a)** Show that the pedal equation of the curve

$x = 2a \cos\theta - 2 \cos 2\theta$, $y = 2a \sin\theta - a \sin 2\theta$ is $9(r^2 - a^2) = 8p^2$

(b) Find the length of the arc of the curve $x = e^{\theta} \sin\theta$, $y = e^{\theta} \cos\theta$

from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

Q.8. **(a)** Find the shortest distance between the straight line joining the points A(3, 2, -4) and B(1, 6, -6) and the straight line joining the points C(-1, 1, -2) and D(-3, 1, -6). Also find equation of the line of shortest distance and coordinates of the feet of the common perpendicular.

(b) Find an equation of the sphere for which the circle

$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y - 4z - 8 = 0$ is a great circle.

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COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

PURE MATHEMATICS, PAPER-II

Part I: Time Allowed: THREE HOURS　　　　Maximum Marks: 100

Note: **(i)** Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.** Paper.

(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** question from **SECTION-B. ALL** question carry **EQUAL** marks.

(iii) Extra attempt of any question or any part of the attempted question will not be considered.

(iv) Use of Calculator is allowed.

SECTION-A

Q.1. **(a)** Let $\ell^p (p \geq 1)$ be the set of all sequences (ζ_j) of complex numbers such that the series $\sum_{j=1}^{n} |\zeta_j|^p$ converges. Let the real valued function $d : \ell^p \times \ell^p \rightarrow R$ be defined by

$$d(x,y) = \left(\sum_{j=1}^{n} |\zeta_j - \eta_j|^p \right)^{1/p}$$

where $x = (\zeta_j)$ and $y = (\eta_j)$. Show that d is a metric on ℓ^p.

(b) If d is the usual metric on R^n (the set of all ordered n-truples of real numbers) then prove that (R^n, d) is a complete metric space.

(c) Prove that the function $f : (X, d_x) \rightarrow (Y, d_y)$ is continuous $\Leftrightarrow f^{-1}(G)$ is closed in X whenever G is closed in Y.

Q.2. **(a)** Prove that there exists no rational number x such that $x^2 = 2$.

(b) Examine the continuity of f at $x = 0$ when

$$f(x) = \begin{cases} x \text{ Sin } \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(c) Find the nth derivative of the function $e^x ln$.

(d) Show that $f(x)=\dfrac{\ell n(x+1)}{x}$ decreases on $]0, \infty[$.

Q.3. **(a)** If $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \dfrac{1}{2}$; find c of the Mean Value Theorem.

(b) Examine the series $\sum_{n=1}^{\infty} \dfrac{n!}{n^2}$ for convergence or divergence.

(c) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{2^n}{n^3}$ Converges or diverges.

(d) Let $f(x) = |x|$. Check the differentiability of f at $x = 0$.

Q.4. **(a)** **If** $u = \text{Sin}^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ **then show that** $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

(b) Find the percentage error in calculating the area of a rectangle when there is error of 2 percent in measuring its sides.

(c) An open rectangular box is to be made from a sheet of cardboard 8dm by 5dm, by cutting equal squares from each corner and turning up the sides. Find the edge of the square which makes the volume maximum.

(d) Find the asymptotes of the curve, $y = \dfrac{x^3+x-2}{x-x^2}$.

Q.5. **(a)** Evaluate the double integral of $F(x, y) = x^2 + xy$, over the triangle with vertices (0, 0), (0, 1) and (1, 1).

(b) Let f be Riemann integrable on $[a, b]$. Prove that $|f|$ is also Riemann integrable on $[a, b]$ and

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\,dx$$

(c) Examine the convergence of the improper integral $\int_0^2 \dfrac{dx}{2x-x^2}$.

SECTION-B

Q.6. **(a)** Solve the equation, $z^2 + (2i - 3)z + 5 - i = 0$

(b) Prove that

$$\text{Cos}^{-1}(\cos\theta + i\sin\theta) = \text{Sin}^{-1}(\sqrt{\sin\theta}) + i\,\ell n(\sqrt{1+\sin\theta} - \sqrt{\sin\theta})$$

(c) If $w = f(z)$ is differentiable then prove that $f(z)$ is continuous.

Q.7. **(a)** Prove that the essential characteristics for a function $f(z)$ to be analytic is that $\frac{\partial f}{\partial \bar{z}} = 0$.

(b) if $u(x, y)$ is a harmonic function then prove that it satisfies the differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$.

(c) Show that the function $f(z) = \cos(z + \frac{1}{z})$ can be expanded as a Laurent's series.

$$f(z) = a_0 + \sum_{n=1}^{\infty} a_n (z^n + \frac{1}{z^n}),$$

where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos(2\cos\theta)\cos n\theta \, d\theta$

Q.8. **(a)** Prove that $\int_{-\infty}^{\infty} \frac{a\cos x + x\sin x}{x^2 + a^2}\,dx = \frac{2\pi}{e^a}, a > 0$

(b) Prove that $\int_0^{\infty} \frac{\sin x}{x}\,dx = \frac{\pi}{2}$

(c) Let $f(z)$ be analytic on a closed contour $C : |z - a| = r$. If $|f(z)| \le M$ then prove that $\left|f^n(a)\right| \le \frac{n!}{r^n} M$.