

FEDERAL PUBLIC SERVICE COMMISSION



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

- NOTE:** (i) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper**.
 (ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B**. All questions carry **EQUAL** marks.
 (iii) **Use of Calculator is allowed.**
 (iv) Extra attempt of any question or any part of the attempted question will not be considered.

SECTION-A

- Q.1. (a)** Find a function ϕ such that $\nabla\phi = \vec{f}$ **(10)**

$$\vec{f} = x\hat{i} + 2y\hat{j} + 2\hat{k}$$

- (b)** Prove that **(10)**

$$\nabla \phi^n = n\phi^{n-1}\nabla \phi$$

- Q.2. (a)** Show that for any vectors \vec{a} and \vec{b} **(10)**

$$\left| \vec{a} + \vec{b} \right|^2 + \left| \vec{a} - \vec{b} \right|^2 = 2 \left(\left| \vec{a} \right|^2 + \left| \vec{b} \right|^2 \right)$$

- (b)** Prove that **(10)**

$$\left(\vec{a} \times \vec{b} \right) \cdot \left(\vec{b} \times \vec{c} \right) \times \left(\vec{c} \times \vec{a} \right) = \left(\vec{a} \cdot \vec{b} \times \vec{c} \right)^2$$

- Q.3. (a)** The greatest result that two forces can have is of magnitude P and the least is of magnitude Q . Show That when they act an angle α their resultant is of magnitude **(10)**

$$\sqrt{P^2 \cos^2 \alpha / 2 + Q^2 \sin^2 \alpha / 2}$$

- (b)** A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the rod **(10)**

is inclined to the wall at an angle $\sin^{-1} \left(\frac{b}{a} \right)^{\frac{1}{3}}$

- Q.4. (a)** Three forces P , Q and R act along the BC , CA and AB respectively of triangle ABC . Prove that if $P \cos A + Q \cos B + R \cos C = 0$, then the line of action of the resultant passes through the circum center of the triangle. **(10)**

- (b)** A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1} \left(\frac{wa}{(W+w)(a+l)} \right)$$

APPLIED MATHS, PAPER-I

- Q.5.** (a) Find the volume $\iint_R (x^3 + 4y) dA$ where R is the region bounded by the parabola $y = x^2$ and the line $y = 2x$. (10)

- (b) Evaluate the following line integral (10)

$$\int_c x^2 dy$$

bonded by the triangle having the vertices $(-1,0)$ to $(2,0)$, and $(1,1)$

SECTION-B

- Q.6.** (a) The position of a particle moving along an ellipse is given by $\vec{r} = a \cos t \hat{i} + b \sin t \hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum or minimum magnitude. (10)

- (b) Prove that the speed at any point of a central orbit is given by: (10)

$$vp = h,$$

When h is the areal speed and p is the perpendicular distance from the centre of force, of the tangent at the point, Find the expression for v when a particle subject to the inverse square law of force describes an ellipse, a parabolic and hyperbolic orbit.

- Q.7.** (a) A particle is moving with the uniform speed v along the curve (10)

$$x^2 y = a \left(x^2 + \frac{a^2}{\sqrt{5}} \right)$$

Show that its acceleration has the maximum value at $\frac{10v^2}{9a}$

- (b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a , whose centre is a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O , show that Y satisfies the equations (10)

$$KY^2 + Y(a^2 - 2hK) + K(h^2 - a^2) = 0,$$

where $K = h + \frac{ga^2}{2v_0^2}$

- Q.8.** (a) Find the tangential and normal components of the acceleration of a particle describing the ellipse (10)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

With uniform speed v when the particle is at $a > b$

- (b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass m lb. moving with velocity v in a direction passing through the centre of gravity. If the bullet is embedded a ft., show that the resistance of (10)

the wood to the bullet, supposed uniform, is $\frac{Mm^2}{2(M+m)ga}$ lb.wt. and that the time of

penetration is $\frac{2a}{v}$ sec., during which time the block will move $\frac{ma}{m+M}$ ft.

FEDERAL PUBLIC SERVICE COMMISSION



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

- NOTE:**
- (i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
 - (ii) Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A** and **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C** **ALL** questions carry **EQUAL** marks.
 - (iii) Extra attempt of any question or any part of the attempted question will not be considered.
 - (iv) **Use of Calculator is allowed.**

SECTION-A

Q.No.1. Solve the following equations:

(a) $\frac{d^3 y}{dx^3} + \frac{dy}{dx} = \sec^2 x$ (10)

(b) $\frac{2dy}{dx} - \frac{x}{y} + x^3 \cos y = 0$ (10)

Q.No.2. (a) Find the power series solution of the differential equation (10)
 $(1 - x^2)y'' - 2xy' + 2y = 0$, about the point $x=0$.

(b) Solve $Z(x+y) \frac{\partial Z}{\partial x} + Z(x-y) \frac{\partial Z}{\partial y} = (x^2 + y^2)$. (10)

Q.No.3. (a) Classify the following equations: (5)

(i) $\frac{\partial^2 Z}{\partial x^2} + x^2 \frac{\partial^2 Z}{\partial y^2} - \frac{1}{x} \frac{\partial Z}{\partial x} = 0$

(ii) $x^2 \frac{\partial^2 Z}{\partial x^2} + 2xy \frac{\partial^2 Z}{\partial x \partial y} + y^2 \frac{\partial^2 Z}{\partial y^2} = 4x^2$

(b) Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $-1 < x < 1, t > 0$ (15)

$u(-1, t) = u(1, t); \frac{\partial u}{\partial x}(-1, t) = \frac{\partial u}{\partial x}(1, t)$ for $t > 0$

$u(x, 0) = x+1, -1 < x < 1$.

SECTION-B

Q.No.4. (a) Highlight the difference between a vector and a tensor. What happens if we permute the subscripts of a tensor? (5)

(b) Transform $g^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1/r^2 \end{pmatrix}$ into Cartesian coordinates. (15)

APPLIED MATHEMATICS, PAPER-II

- Q.No.5.** (a) Workout the Christoffel symbols for the metric tensor $g_{ab} = \begin{pmatrix} a^2 & 0 \\ 0 & a^2 \sin^2 \theta \end{pmatrix}$ (10)
- (b) Workout the two dimensional metric tensor for the coordinates p and q given by (10)
- $$p = (xy)^{\frac{1}{3}}, q = \left(\frac{x^2}{y}\right)^{\frac{1}{3}}$$

SECTION-C

- Q.No.6.** (a) Solve the following system of equations by Jacobi iteration method: (10)
- $$10x + y - 2z = 7.74$$
- $$x + 12y + 3z = 39.66$$
- $$3x + 4y + 15z = 54.8$$
- (b) Solve $\sin x = 1 + x^3$ Using Newton-Raphson method. (10)
- Q.No.7.** (a) Find the root of $xe^x = 3$ by regular falsi method correct to three decimal places. (10)
- (b) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ using (5+5) (10)
- (i) Trapezoidal rule and
- (ii) Simpson's rule.
- Q.No.8.** (a) Find the real root of the equation $\cos x = 3x - 1$ correct to seven decimal places by the iterative method. (10)
- (b) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given below: (10)

X	5	6	9	11
Y	12	13	14	16
