

FEDERAL PUBLIC SERVICE COMMISSION



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

- NOTE:** (i) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B**. All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

- Q.1. (a) Find the divergence and curl \vec{f} If $\vec{f} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + (x^2y + 3z^2)\hat{k}$ (10)
(b) Also find a function ϕ such that $\nabla\phi = \vec{f}$ (10)
- Q.2. (a) Find the volume $\iint_R xy \, dA$ where R is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. (10)
(b) Evaluate the following line intergral: (10)
 $\int_c y^2 dx + xdy$ where $c = c_1$ is the line segment joining the points (-5, -3) to (0, 2), and $c = c_2$ is the arc of the parabola $x = 4 - y^2$.
- Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is (10)
 $\sqrt{P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C}$
(b) A hemispherical shell rests on a rough inclined plane whose angle of friction is λ . Show that the inclination of the plane base to the horizontal cannot be greater than $\arcsin(2 \sin \lambda)$ (10)
- Q.4. (a) A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart and in the same horizontal line. Show that if (10)
 $\frac{\theta}{\sqrt{2}} < b < a$, a non symmetric position of equilibrium is possible in which $b(\sin \theta + \cos \theta) = a$
(b) Find the centre of mass of a semi circular lamina of radius a whose density varies as the square of the distance from the centre. (10)

APPLIED MATHEMATICS, PAPER-I

Q.5. (a) Evaluate the integral $\int_0^1 \int_{x^2}^x (x^2 + y^2) dy dx$ (10)

also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis (10)
 $f(x, y) = 4xz^3 - 3x^2y^2z, P = (2, -1, 2)$

SECTION – B

Q.6. (a) A particle is moving along the parabola $x^2 = 4ay$ with constant speed v. Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5a}$ (10)

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time t, if it starts from rest at $t = 0$ and is subject to an acceleration $t^2 + \sin t + e^t$ (10)

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from x – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then (10)

$$y = a \cosh nx + b \sinh nx$$

where a and b are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration x^3 , where x is the distance of the particle from a fixed point O on the line, if it starts at $t = 0$ from a point $x = c$ with the velocity $c^2 / \sqrt{2}$. (10)

Q.8. (a) A battleship is steaming ahead with speed V and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation α . If v_0 is the speed of projection (relative to the gun) show that the range is $\frac{2v_0}{g} \sin \alpha (v_0 \cos \alpha - V)$ (10)

(b) Show that the law of force towards the pole of a particle describing the survey $r^n = a^n \cos n\theta$ (10)
is given by $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$ where h is a constant.

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APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B**. All questions carry equal marks.
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SECTION - A

Q.1. (a) Solve by method of variation of parameter (10)

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \ln x$$

(b) Solve first order non-linear differential equation (10)

$$x \frac{dy}{dx} + y = y^2 \ln x$$

Q.2. (a) Solve (10)

$$c^2 u_{xx} = u_{tt}.$$

$$u(0, t) = 0$$

$$u(l, t) = 0$$

$$u(x, 0) = \lambda \sin\left(\frac{\pi}{l} x\right)$$

$$u_t(x, 0) = 0$$

(b) Solve (10)

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$$

Q.3. (a) Work out the two dimensional metric tensor for the coordinates p and q given by (10)

$$p = (xy)^{\frac{1}{3}}, q = (x^2 / y)^{\frac{1}{3}}$$

(b) Prove that (10)

$$\Gamma_{ab}^d = \frac{1}{2} g^{dc} \left(g_{ac,b} + g_{bc,a} - g_{ab,c} \right)$$

APPLIED MATHEMATICS, PAPER-II

Q.4. (a) Work out the Christoffel symbols for the following metric tensor (10)

$$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

(b) Work out the covariant derivative of the tensor with components (10)

$$\begin{pmatrix} r \cos \theta & ar \sin \varphi & ar \\ \sin \theta \sin \varphi & a \sin \theta \cos \varphi & a \\ \cos \varphi & a \sin \varphi & 0 \end{pmatrix}$$

Q.5. (a) Find recurrence relations and power series solution of $(x-3)y' + 2y = 0$ (10)

(b) Solve the Cauchy Euler's equation $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$ (10)

(10)

SECTION – B

Q.6. (a) Find the positive solution of the following equation by Newton Raphson method (10)

$$2 \sin x = x$$

(b) Solve the following system by Jacobi method: (10)

$$10x_1 - 8x_2 = -6$$

$$-8x_1 + 10x_2 - x_3 = 9$$

$$-x_2 + 10x_3 = 28$$

Q.7. (a) Evaluate the following by using the trapezoidal rule. (10)

$$\int_0^1 (x+1) dx$$

(b) Evaluate the following integral by using Simpson's rule (10)

$$\int_0^4 e^x dx$$

Q.8. (a) Solve the following equation by regular falsi method: (10)

$$2x^3 + x - 2 = 0$$

(b) Calculate the Lagrange interpolating polynomial using the following table: (10)

x	0	1	2
f(x)	1	0	-1

also calculate f (0.5).
